LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

**M.Sc.** DEGREE EXAMINATION - **STATISTICS**

THIRD SEMESTER – NOVEMBER 2010

# ST 3812 / 3809 - STOCHASTIC PROCESSES

Date : 01-11-10 Dept. No. Max. : 100 Marks

Time : 9:00 - 12:00

**SECTION - A**

**Answer all the questions (2x10=20)**

1)What are the different types of classification of a stochastic process ?

2) Define a recurrent state and mean recurrence time of a state.

3) Define a process with independent increments . When it is said to be stationary?

4) Customers enter a store according to a Poisson process of rate λ = 6 per hour.

Suppose it is known that a single customer entered during the first hour .

What is the conditional probability that this person had entered during the first fifteen minutes?

5) Define a Martingale.

6) Define convolution of two independent random variables.

7) Define excess life and current life of a renewal process.

8) Given X1, X2, .... are i . i .d random variables . Obtain E[ X1+ X2+...+XN ] where N is also

a random variable independent of X1,X2…

9) What are the postulates in a pure death process ?

10) What is the distribution of waiting time for n events in a Poisson process ?

**SECTION - B**

**Answer 5 questions only. 5 x 8 = 40**

11) Explain how a discrete queueing model can be viewed as a Markov chain .

12) For a Markov chain with states 0,1,2 and initial distribution

P[X0 = i] = , i=0,1,2 and

P =,

obtain

i) P[X3 =1,X2 =1,X1 =1,X0 =2]

ii) P[X2=2]

iii) P[X3 =1,X2 =1│X1 =1,X0 =2] (2+4+2)

13) Obtain the differential equations for a pure birth process and obtain Pn(t) .

14) Let X(t) be a Poisson process with parameter λ. Suppose each arrival is registered with

probability p ,independent of other arrivals. Let Y(t) be the process of registered arrivals .

Prove that Y(t) is a Poisson process with parameter λp

15) Show that the renewal function satisfies M(t) = F(t) +

16) Consider a Markov chain with states 0, 1 and

P=

Show that Pn =

17) Let {Xn} be a super martingale with respect to {Yn}.Show that

i) E[Xn+k / Y0,Y1,....,Yn ] ≤ Xn

ii) E[Xn ] E [Xk]  E[X0] , 0  k  n .

18) Let the distribution of number of offsprings be Pk=b (1-b)k , k = 0,1,2...

Obtain the probability of extinction.

**SECTION - C**

**Answer any TWO questions: (2X20=40)**

19) a) In a positive recurrent aperiodic class with states 0,1,2,3.... and

= , show that = and =1 and π’s are uniquely determined.

b) Consider the Markov chain with states 0,1,2,3 and transition probability matrix

P=

using first step analysis obtain

Ui = P[XT=0 / X0= i] ,i = 1,2

Vi = E[T/X0 = i] ,i = 1,2 where T is the time for absorption . (10+10)

20) i. State the postulates of birth and death process.

ii. Derive the backward and forward differential equations.

iii. Obtain the stationary distribution.

iv. Derive the stationary distribution in the case of queue with a single server. (4+ 8+5+3)

21) a) For a renewal process with life time density f(x) = ,x≥0 , obtain the renewal function.

b) Let Y0,Y1,....... represent a Markov chain with transition probability matrix P. Let f be a

bounded right regular sequence (i.e) f(i)=

If Xn=f( Yn). Show that Xn is a Martingale .

c) Obtain the generating function relations for a branching process. (8+4+8)

22) a) In a Markov chain if i↔j show that d(i) = d(j) .

b) A Markov chain on states {0,1,2,3,4,5} has transition probability matrix



Obtain i=0,1,2,3,4,5. (8+12)

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